PD – 093

II Year M.Sc. (DCC) Examination, January 2018
(Y2K13 Scheme) (Fresh and Repeaters)
MATHEMATICS
M 205 A : Mathematical Methods, Modelling and Simulation

Time : 3 Hours Max. Marks : 80

Instructions *: 1) Answer* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1 a) Obtain the solution of the IBVP.

$$\frac{\partial^2 u}{\partial t^2} - 9\frac{\partial^2 u}{\partial x^2} = \sin(\omega t)\,;\ 0 < x < \infty,\ t \geq 0$$

subject to

u (0, t) = 0; u is bounded as $x \to \infty$,

$$\frac{\partial u}{\partial t}(x, 0) = u(x, 0) = 0\,,$$

By the Laplace transform method. **8**

b) Solve the following Fredholm integral equation of second kind :

$$u(x) = \lambda \int_0^{2\pi} \sin(x + t)\, u(t)\, dt.$$ **8**

2. a) Find the asymptotic expansion of

$$I(x) = \int_0^{\infty} \frac{e^{-t}}{1 + xt}\, dt \text{ as } x \to 0.$$ **8**

b) Use Laplace method, obtain the asymptotic expansion of

$$I(x) = \int_0^{3} e^{x} \cosh^2 t\, dt \text{ as } x \to \infty.$$ **8**

3. a) With the help of Watson's lemma solve

$$I(x) = \int_1^{\infty} (s^2 - 1)^{-\frac{1}{2}} e^{-xs}\, ds \text{ as } x \to \infty.$$ **8**

b) Find the leading order term of

$$I(x) = \int_0^1 e^{ix(t - \sin t)}\, dt.$$

by stationary phase method. **8**

4. Apply appropriate perturbation method to solve the following problems :

a) $\varepsilon y'' + a(x)\, y' + b(x)\, y(x) = 0,\ y(0) = A,\ y(1) = B,$ **8**

b) $u''(x) + u(x) = \varepsilon u^2(x),\ u(0) = a,\ u'(0) = 0$. **8**

5. a) From a real world problem to its mathematical model, explain the various steps involved. **8**

b) Derive any one population growth model and discuss its solution. **8**

6. a) Find travelling wave solution of the Burger equation. **8**

b) Explain the effects of convection and diffusion in the formation of shocks. **8**

7. a) Define ground water and explain dam seepage in ground water flow. **8**

b) Explain about the validity of Darcy model. **8**

8. Explaining the terminology used in atmospheric pollution, derive the conservation of mass and linear momentum for a simple-illustrative turbulent flow. **16**
